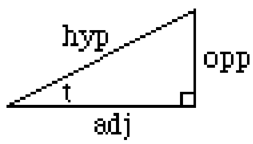
MA-161

Precalculus Formula Sheet and Trig Helper

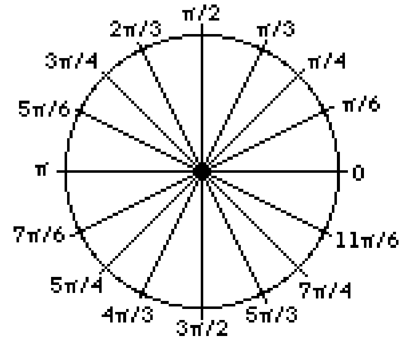
$$\frac{\text{Radian Measure}}{\pi} = \frac{\text{Degree Measure}}{180}$$



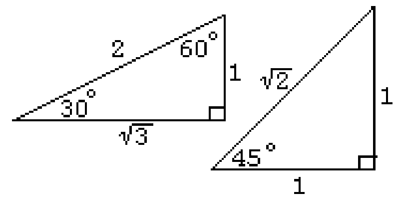
$$\sin t = \frac{\text{opp}}{\text{hyp}}$$

$$\cos t = \frac{\text{adj}}{\text{hyp}}$$

$$\tan t = \frac{\text{opp}}{\text{adj}}$$



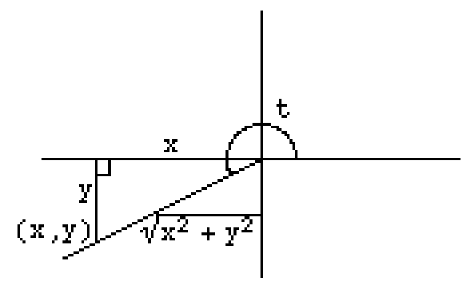
	0° (0 rads)	30° (p/6)	45° (p/4)	60° (p/3)	90° (p/2)
sine	0	1/2	$\sqrt{2}/2$	$\sqrt{3}/2$	1
cosine	1	$\sqrt{3}/2$	$\sqrt{2}/2$	1/2	0
tangent	0	$\sqrt{3}/3$	1	$\sqrt{3}$	Undef



General Definition of Trig Functions

For any real number, t, construct an angle in standard position with radian measure t. Choose an arbitrary point (x,y) on the terminal side. Then

$\sin t = \dfrac{y}{\sqrt{x^2 + y^2}}$ $\cos t = \dfrac{x}{\sqrt{x^2 + y^2}}$ $\tan t = \dfrac{y}{x}$



Trig Formulas and Identities

$\csc t = 1/\sin t$ $\sec t = 1/\cos t$ $\cot t = 1/\tan t$ $\tan t = \sin t/\cos t$

$\sin(-t) = -\sin t$ $\cos(-t) = \cos t$ $\sin^2 t + \cos^2 t = 1$ $\tan^2 t + 1 = \sec^2 t$

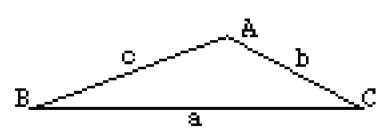
$\sin(t \pm u) = (\sin t)(\cos u) \pm (\cos t)(\sin u)$ $\sin(2t) = 2(\sin t)(\cos t)$

$\cos(t \pm u) = (\cos t)(\cos u) \mp (\sin t)(\sin u)$ $\cos(2t) = \cos^2 t - \sin^2 t = 2\cos^2 t - 1 = 1 - 2\sin^2 t$

$\sin(t/2) = \pm\sqrt{(1-\cos t)/2}$ $\cos(t/2) = \pm\sqrt{(1+\cos t)/2}$

Cosine Law For DABC with sides of lengths shown:

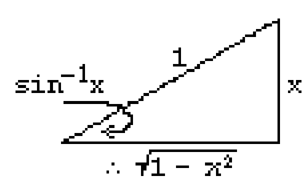
$$a^2 = b^2 + c^2 - 2bc\cdot\cos A$$



Inverse Trig Functions (The definitions are similar so only $\sin^{-1}$ is given.)

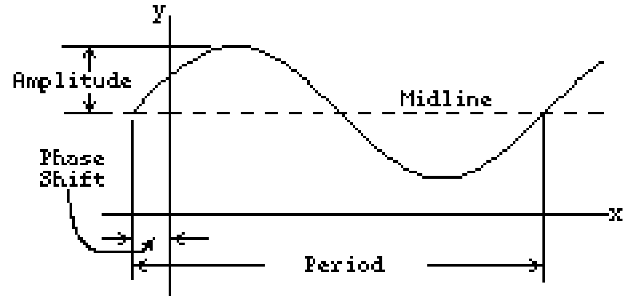
$\sin^{-1} x$ is the angle between –p/2 and p/2 inclusive whose sine is x.

Facts: $\sin(\sin^{-1} x) = x$ $\sin^{-1}(\sin x) = x$ $\cos(\sin^{-1} x) = \sqrt{1 - x^2}$



(To do the last one, draw a right triangle, one of whose angles is $\sin^{-1} x$. Label the lengths of the sides using the meaning of $\sin^{-1} x$ and Pythagoras. Then use the triangle to write the cosine of the angle.)

Graphing Formulas



For the equation $y = d + a\cdot\sin(bx - c)$ where $b > 0$:

-- The midline is the line $y = d$;

-- The amplitude is $|a|$. If $a < 0$, the graph is reflected about the midline;

-- The period is $2p/b$; and

-- The phase shift is c/b.

A similar set of rules holds for $y = d + a\cdot\cos(bx - c)$.

Analytic Geometry Formulas

Circle: $(x - h)^2 + (y - k)^2 = r^2$ or $x^2 + y^2 + Dx + Ey + F = 0$

Parabola: $y = A(x - h)^2 + k$ where $A = 1/[4\cdot(\text{dist from vertex to fucus})]$ or $x^2 + Dx + Ey + F = 0$

Ellipse: $\frac{(x-h)^2}{r^2} + \frac{(y-k)^2}{s^2} = 1$

Quadratic Formula The solutions to $ax^2 + bx + c = 0$ are $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Factorization Formula If n is a natural number, then

$$a^n - b^n = (a - b)(a^{n-1} + a^{n-2}b + a^{n-3}b^2 + \ldots + ab^{n-2} + b^{n-1})$$

Series Formulas

(A.S. Formula) $1 + 2 + 3 + \ldots + w = \frac{w\cdot(w+1)}{2}$

(G.S. Formula) $1 + r + r^2 + \ldots + r^w = \frac{r^{w+1} - 1}{r - 1} = \frac{1 - r^{w+1}}{1 - r}$.

If $-1 < r < 1$, then $a + ar + ar^2 + ar^3 + \ldots = \frac{a}{1 - r}$.

$1^2 + 2^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$ $1^3 + 2^3 + \ldots + n^3 = \left(\frac{n(n+1)}{2}\right)^2$

Miscellaneous Formulas

Exponents and Logs: $r^{\log_r s} = s$ Change of Base: $\log_r x = \frac{\log_s x}{\log_s r}$

Compound Interest: $A = P\left(1 + \left(\frac{r}{n}\right)\right)^{nt} = P(1 + (\text{interest rate per period}))^{\text{number of periods}}$

Continuously Compounded Interest: $A = Pe^{rt}$.

Exponential Growth or Decay: $y = a\cdot b^x$ or $y = a\cdot e^{bx}$ where a and b are constants.

Differentiation Formulas **(Don't forget the Product, Quotient, and Chain Rules!)**

$\frac{d}{dx}(u^n) = n\cdot u^{n-1}\frac{du}{dx}$ $\frac{d}{dx}(\sin u) = \cos u\frac{du}{dx}$ $\frac{d}{dx}(\cos u) = -\sin u\frac{du}{dx}$ $\frac{d}{dx}(\tan u) = \sec^2 u\frac{du}{dx}$

$\frac{d}{dx}(e^u) = e^u\frac{du}{dx}$ $\frac{d}{dx}(\ln u) = \frac{1}{u}\cdot\frac{du}{dx}$ $\frac{d}{dx}(\arcsin u) = \frac{1}{\sqrt{1-u^2}}\cdot\frac{du}{dx}$ $\frac{d}{dx}(\arctan u) = \frac{1}{1+u^2}\cdot\frac{du}{dx}$