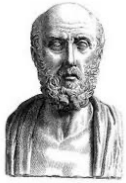


Euclid's Proof of the Pythagorean Theorem

- ▼ 1. Greece flourished in the 150 years between Hippocrates & Euclid.
 - ▼ a. Plato (427 BC) and his student Eudoxus (408 BC) led the way
 - i. Many sophists (clever know-it-alls who taught for money) wandered the land - some brilliant, some quacks
 - ii. **Socrates'** disciple was **Plato**, and his student was **Aristotle**
 - ▼ b. Plato traveled until age 40, then founded the Academy in Athens. The intellectual center of the world, headquarters of Greek wisdom.
 - i. Learned from the Pythagoreans in Italy, visited Egypt, etc. Sold as a slave, but then ransomed by his friends.
 - ii. His school is the spiritual ancestor of our institution of higher learning, the land had belonged to the hero Akademos.
 - iii. As was custom, to establish legitimacy the Academy was a "religious brotherhood" worshiping the Muses
 - iv. For 900 years it stood as intellectual center of Greece until 529 AD, was closed b/c it was "pagan" and of "perverse learning"
 - ▼ c. Math was king, Geometry was required for entrance.
 - i. Plato disliked "applied mathematics" and geometric instruments, embracing only the straightedge and compass
 - ▼ d. **Eudoxus** was poor, commuted in to Athens, and had two main pieces of mathematics, was 2nd only to Archimedes
 - ▼ i. Theory of proportion
 - 1. Many proofs of "similar triangles have equal ratios of corresponding sides" required the (false) commensurability of numbers.
 - 2. Eudoxus' devised a proof that did not appeal to commensurability.
 - 3. His proof is found in Book V of the Elements (Euclid didn't prove everything there.)
 - ▼ ii. Method of Exhaustion
 - 1. Main idea of limits/calculus.
 - 2. Archimedes used/credited Eudoxus' idea to find the area of a circle.
 - ▼ e. Alexander the Great & Alexandria
 - i. 332 BC Alex (20 y/o from Macedonia) conquered Egypt, established a new city Alexandria
 - ii. Alexandria grew while Alex "conquered the world" and became the intellectual center
 - iii. 500 K people and 600K (really?) Papyrus rods, Alexandrian Library & Museum > Academy,
 - iv. The Museum (the oldest university) was heaven on earth for thinkers, food, space community.
 - v. Alex conquered Greece, Egypt, most of Asia Minor, and died trying to capture India.
 - vi. Alexandria remained central & powerful for 300 years - the Hellenistic (Greek-like) age, until the Roman Empire.
 - vii. 300 - 100 BC is the 2nd most mathematically productive era ever. #1 = Kepler - Gauss (1600 -1850)
 - viii. Archimedes, Eratosthenes, Apollonius Pappus, Claudius Ptolemy, Diophantus & Euclid all worked here at some time.

greek philosophers



Hippocrates - 430 BCE



Sophist

Greece flourished during this period

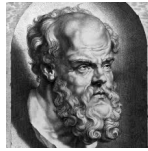


Euclid - 300 BCE



A Diagram and a step,
not a diagram $\frac{1}{2}$ a coin.

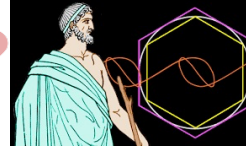
In ancient Greece,
math was king
geometry was
required for
entrance



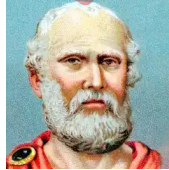
Socrates



Pythagoras



Eudoxus - 408 BCE



Plato - 427 BCE



Aristotle

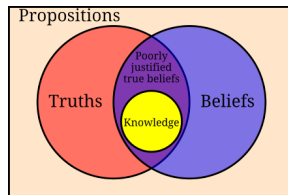
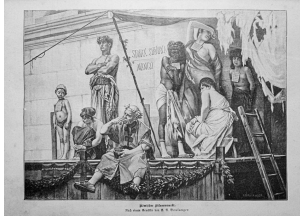
spiritual ancestor
of the University



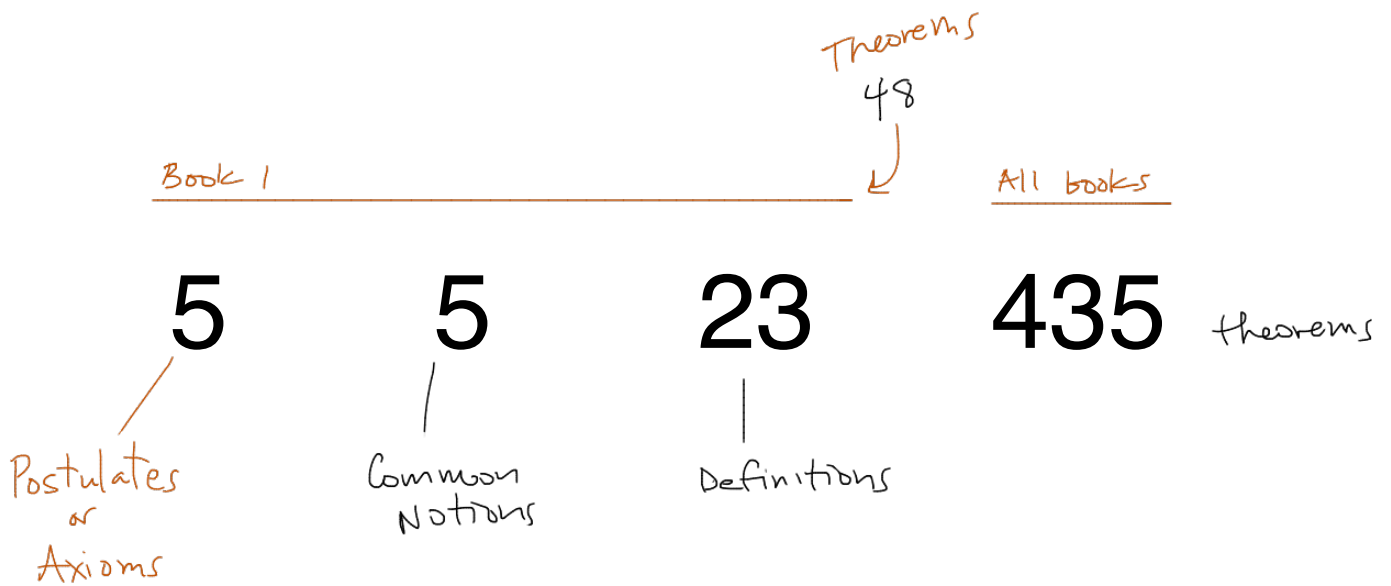
Academy

lasted 900
years, until
529 AD

closed due to: pagan, perverse learning



Euclid's Elements : 13 books



Common Notions

Common notion 1.

Things which equal the same thing also equal one another.

Common notion 2.

If equals are added to equals, then the wholes are equal.

Common notion 3.

If equals are subtracted from equals, then the remainders are equal.

Common notion 4.

Things which coincide with one another equal one another.

Common notion 5.

The whole is greater than the part.

Outline of Thales

Thales of Miletus - 625 BC

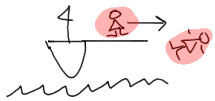
- ▼ 1. The First Mathematician
 - a. Required proof
 - b. Oldest "proof" is his
- ▼ 2. Began the stereotype of the absent-minded genius
 - a. Never married
 - b. Was one "well" of a mathematician
- 3. "The most difficult thing to know in life is yourself."
- ▼ 4. Proved :
 - a. Angle inscribed in a semi-circle is a right angle.
 - b. Base angles of an isosceles triangle are equal.
 - c. If two straight lines intersect, the opposite angles are equal.
 - d. Angle sum of a triangle is two right angles.
- 5. Many of his theorems were perhaps known to the Egyptians, and conventional history seeks to look for some individual to whom the "miracle" can be ascribed - Thales is the natural candidate. He certainly contributed much to the rational organization of geometry (the deductive method).
- 6. The orderly development of theorems by rigorous proof was new and unique to Greek mathematics.



Thales

- 1. Visited Egypt, made indirect measurement of the height of the Great Pyramid by means of shadows.
- 2. Predicted solar eclipse in 585 BC, or did he?
- 3. Perhaps taught Pythagoras everything he knew.

Pythagoreans $\frac{1}{2} \sqrt{2} \neq \frac{a}{b}$



Next let's see why $\sqrt{2}$ is irrational.

Lemma: If a square number is even, then its square root is even too.

(ex) 16 is square, even, $\frac{1}{2} \sqrt{16} = 4$ — even too! if
 49 is square, not even, so skip —
 $16 = 4 \cdot 4$
 $\quad \quad \quad \parallel$
 $\quad \quad \quad 2n$
 $\quad \quad \quad \parallel$
 $\quad \quad \quad 2 \cdot 8$

$a^2 = 2n \Rightarrow$ means there is one "2"
 $= a \cdot a \Rightarrow$ since it's a square all "atoms" appear twice
 $= \underline{2k} \cdot \underline{2k}$
 $\Rightarrow \sqrt{a^2} = 2k$

PRIME DECOMPOSITION: every whole # decomposes into a unique set of primes

$$\begin{array}{l|l} 54 = 2 \cdot 27 & 36 = \underline{6} \cdot \underline{6} \\ = 2 \cdot 3 \cdot 9 & = 2 \cdot 3 \cdot 2 \cdot 3 \\ = 2 \cdot 3 \cdot 3 \cdot 3 & = 4 \cdot 3 \cdot 3 \\ & \quad \quad \quad \parallel \end{array}$$

theorem: $\sqrt{2}$ is not rational

proof:

Reducto ad absurdum

(Assume $\sqrt{2}$ is rational, thus $\sqrt{2} = \frac{a}{b}$

where $\frac{a}{b}$ is a reduced fraction of whole #'s.

(ex) $\frac{12}{8}$ is not reduced (b/c = $\frac{6}{4}$ or $\frac{3}{2}$)

↑
reduced!

① $\sqrt{2} = \frac{a}{b}$ ← square both sides

② $2 = \frac{a}{b} \cdot \frac{a}{b} = \frac{a^2}{b^2} = \frac{2}{1}$

(idea:

$\frac{12}{8} = \frac{3}{2}$

↓ cross mult.

$12 \cdot 2 = 3 \cdot 8$

③ $a^2 = 2b^2$ cross-mult.

④ a^2 is even

⑤ a must be even too!

⑥ so $a = 2x$

⑦ but combining ⑥ w/ ③ gives

$a^2 = 2b^2$

$(2x)^2 = 2b^2$

~~$2x \cdot 2x = 2b^2$~~

$2x^2 = b^2$

÷ by 2
both sides

⑧ so b^2 is even

⑨ now b must be even

⑩ but $\frac{a}{b}$ was supposed to be a reduced fraction

⇒ no common factors

(but we've arrived at a lie suggesting both have 2 as a factor)

contradiction (X)

thus $\sqrt{2} \neq \frac{a}{b}$ (this cost some poor Pythagorean his life)