Math 163 - Calculus - Exam 3 - Practice **Name:** _____

October 31, 2026

On the exam you must show your work to receive full credit.

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1. Find the radius of convergence and the interval of convergence of

$$\sum_{n=1}^{+\infty} \frac{(2x)^n}{n}$$

2. Find the radius of convergence of

$$\sum_{k=1}^{+\infty} \frac{(k!)^2}{(2k)!}\, x^k$$

3. Use $\dfrac{1}{1-x} = \displaystyle\sum_{n=0}^{+\infty} x^n$ to find a power series for the function below, centered at $a = 0$, and find its interval of convergence.

$$f(x) = \frac{x^2}{1-3x^2}$$

4. Find a power series for $f(x) = \dfrac{1}{6-x}$ centered at $a = 5$, and find its interval of convergence.

5. Use partial fraction decomposition to find a power series for

$$f(x) = \frac{2}{(x-3)(x+1)}$$

centered at $a = 0$.

6. Differentiate $f(x) = \dfrac{1}{1-x} = \displaystyle\sum_{n=0}^{+\infty} x^n$ term by term, and use the result to find the exact value of

$$\sum_{n=1}^{+\infty} \frac{n}{3^n}$$

7. Use term-by-term integration to find a power series for $f(x) = \ln(4 - x)$ centered at $a = 0$. What is its radius of convergence?

8. Find the Taylor polynomial of degree 2 for $f(x) = \cos(3x)$ centered at $a = \pi$.

9.(9.1) Find the Taylor series of $f(x) = e^x$ centered at $a = -1$.

(9.2) Find the Taylor series of $f(x) = \dfrac{1}{x}$ centered at $a = -1$, and find its interval of convergence.

10. Let $f(x) = (1 + x^2)^{-1/3}$.

(10.1) Use the binomial series to write the Maclaurin series of f in Σ-notation.

(10.2) Find the sixth Maclaurin polynomial $P_6(x)$ for f.

11. Find a power series for

$$F(x) = \int_0^x e^{-t^2}\, dt$$

12. Find the first three nonzero terms of the Maclaurin series for $f(x) = \ln(\cos x)$.